Transport theory with nonlocal corrections

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Abstract

A kinetic equation which combines the quasiparticle drift of Landau's equation with a dissipation governed by a nonlocal and noninstant scattering integral in the spirit of Snider's equation for gases is derived. Consequent balance equations for the density, momentum and energy include quasiparticle contributions and the second order quantum virial corrections and are proven to be consistent with conservation laws.

The very basic idea of the Boltzmann equation (BE), to balance the drift of particles with dissipation, is used both in gases, plasmas and condensed systems like metals or nuclei. In both fields, the BE allows for a number of improvements which make it possible to describe phenomena far beyond the range of validity of the original BE. In these improvements the theory of gases differs from theory of condensed systems. In theory of gases, the focus was on so called virial corrections that take into account a finite volume of molecules, e.g. Enskog included space non-locality of binary collisions [1]. In the theory of condensed systems, modifications of the BE are determined by the quantum mechanical statistics. A headway in this field is covered by the Landau concept of quasiparticles [2]. There are three major modifications: the Pauli blocking of scattering channels; underlying quantum mechanical dynamics of collisions; and quasiparticle renormalization of a single-particle-like dispersion relation. However, the scattering integral of the BE remains local in space and time. In other words, the Landau theory does not include a quantum mechanical analogy of virial corrections. The missing link of two major streams in transport theory is clearly formulated by Laloë and Mullin [3] in their comments on Snider's equation. Our aim is to fill this gap. Briefly, here we derive a transport equation that includes quasiparticle renormalizations in the standard form of Landau's theory and virial corrections in the form similar to the theory of gases. "Particle diameters" and other non-localities of the scattering integral are given in form of derivatives of phase shift in binary collisions [4,5].

A convenient starting point to derive various corrections to the BE is the quasiparticle transport equation first obtained by Kadanoff and Baym

$$\frac{\partial f}{\partial t}+\frac{\partial \varepsilon}{\partial k}\frac{\partial f}{\partial r}-\frac{\partial \varepsilon}{\partial r}\frac{\partial f}{\partial k}=z(1-f)\Sigma_\varepsilon^<-zf\Sigma_\varepsilon^>. \tag{1}$$

Here, quasiparticle distribution f, quasiparticle energy ε and wave-function renormalization z are functions of time t, coordinate r, momentum k and isospin a. The self-energy $\Sigma^{>,<}$ is moreover a function of energy ω, however it enters the transport equation only by its value at pole $\omega=\varepsilon$. The drift terms in the l.h.s of (1) have the standard form of the BE except that the single-particle-like energy ε is renormalized. This is exactly the form of drift visualized by Landau. The scattering integral in the r.h.s. of (1) is, however, more general than expected by Landau, in particular, it includes virial corrections which emerge for complex self-energies [6]. The self-energy we discuss is constructed from a two-particle T-matrix in the Bethe-Goldstone approximation (for simplicity, we have left aside the exchange term) $\Sigma^<(1,2)=T^R(1,\bar 3;\bar 5,\bar 6)T^A(\bar 7,\bar 8;2,\bar 4)G^>(\bar 4,\bar 3)G^<(\bar 5,\bar 7)G^<(\bar 6,\bar 8)$, which is known to include non-trivial virial corrections [7]. Here, G's are single-particle Green's functions, numbers are cumulative variables, $1\equiv(t,r,a)$, time, coordinate and isospin. Bars denote internal variables that are integrated over. The self-energy as a functional of Green's functions $\Sigma[G]$ is converted into the scattering integral $\Sigma_\varepsilon[f]$ via the quasiparticle approximation $G^>(\omega,k,r,t,a)=(1-f(k,r,t,a))2\pi\delta(\omega-\varepsilon(k,r,t,a))$ and $G^<(\omega,k,r,t,a)=f(k,r,t,a)2\pi\delta(\omega-\varepsilon(k,r,t,a))$. Omitting gradient contributions to collisions one simplifies the scattering integral, but on cost of virial corrections. Indeed, the space and time non-locality of the scattering integral is washed out in absence of gradients. To obtain the scattering integral with virial corrections we linearize all functions in a vicinity of (r,t) using r^i-r and t^1-t as small parameters to second order. Then the scattering integral of equation (1) results

$$\begin{aligned}&\sum_b\int\frac{dp}{(2\pi)^3}\frac{dq}{(2\pi)^3}2\pi\delta\left(\varepsilon_a^0+\varepsilon_b^3-\varepsilon_a^1-\varepsilon_b^2+2\Delta_E\right)\\&\times|T|^2\left(\varepsilon_a^0+\varepsilon_b^3-\Delta_E,k-\frac{\Delta_K}{2},p-\frac{\Delta_K}{2},q,t-\frac{1}{2}\Delta_t,r-\Delta_r\right)\\&\times\Big[f_a^1f_b^2(1-f_a^0)(1-f_b^3)-(1-f_a^1)(1-f_b^2)f_a^0f_b^3\Big].\end{aligned} \tag{2}$$

Here, $\upsilon_a^0=(k,r,t,a)$, $\upsilon_a^1=(k-q-\Delta_K,r-\Delta_3,t-\Delta_t,a)$, $\upsilon_b^2=(p+q-\Delta_K,r-\Delta_4,t-\Delta_t,b)$, $\upsilon_b^3=(p,r-\Delta_2,t,b)$, and $\varepsilon_a^i=\varepsilon(\upsilon_a^i)$ and $f_a^i=f(\upsilon_a^i)$. One has to keep in mind that form (2) holds only up to its linear expansion in Δ's. All

Δ's are given by derivatives of the phase shift $\phi = \text{Im}\ \ln T^R_{\text{sc}}(\Omega, k, p, q, t, r)$,

$$
\begin{aligned}
\Delta_t &= \left.\frac{\partial \phi}{\partial \Omega}\right|_{\varepsilon_1+\varepsilon_2} \qquad & \Delta_2 &= \left(\frac{\partial \phi}{\partial p} - \frac{\partial \phi}{\partial q} - \frac{\partial \phi}{\partial k}\right)_{\varepsilon_1+\varepsilon_2} \\
\Delta_E &= -\frac{1}{2}\left.\frac{\partial \phi}{\partial t}\right|_{\varepsilon_1+\varepsilon_2} \qquad & \Delta_3 &= -\left.\frac{\partial \phi}{\partial k}\right|_{\varepsilon_1+\varepsilon_2} \\
\Delta_K &= \frac{1}{2}\left.\frac{\partial \phi}{\partial r}\right|_{\varepsilon_1+\varepsilon_2} \qquad & \Delta_4 &= -\left(\frac{\partial \phi}{\partial k} + \frac{\partial \phi}{\partial q}\right)_{\varepsilon_1+\varepsilon_2}
\end{aligned}
\tag{3}
$$

and $\Delta_r = \frac{1}{4}(\Delta_2 + \Delta_3 + \Delta_4)$. After derivatives, Δ's are evaluated at the energy shell $\Omega \to \varepsilon_1 + \varepsilon_2$ The Δ's are effective shifts and they represent mean values of various non-localities of the scattering integral. These shifts enter the scattering integral in form known from theory of gases [1], however, the set of shifts is larger than the one intuitively expected. The full set (3) is necessary to guarantee gauge invariance. One can see that sending all Δ's to zero, the scattering integral (2) simplifies to the one used in the BE for quasiparticles. The scattering integral is interpreted as collision at time t and coordinate r in which two particles (holes) a and b of momenta k and p scatter into final states of momenta $k - q$ and $p + q$. This interpretation is correct for the weak-coupling limit $T^R \approx V$, where the phase shift in dissipative channels vanishes, $\phi = 0$, and no virial corrections appear. With nontrivial Δ's, the interpretation has to be slightly modified due to finite collision duration and finite "particle diameters". For instant potential, the particles a and b enter the collision at the same time instant (there is no time shift between arguments v_a^0 and v_b^3) and leave the collision together (there is no time shift between v_a^1 and v_b^2). The only time shift Δ_t is between the beginning and the end of collision. This time shift is just the collision delay discussed by Danielewicz and Pratt [8]. Due to the finite duration of the collision, the pair of particles a and b can gain an energy $2\Delta_\omega$ from external fields. The momentum shift $2\Delta_k$ describes an acceleration the pair of particles picks up during their correlated motion.

With respect to a general form of the transport equation we have already fulfilled our task: the quasiparticle transport equation (1) with the non-local scattering integral (2) is our final result. This transport equation has complicated self-consistent structure: (i) quasiparticle energy depends on distributions via real part of self-energy, (ii) scattering rate depends on distributions via Pauli blocking of two-particle propagation in T-matrix, (iii) Δ's depend on distributions also due to Pauli blocking. The same complexity one meets for the quasiparticle BE, except for neglected Δ's. In fact, Δ's do not represent much of additional work as the T-matrix has to be evaluated within the BE anyway. To summarize, we have derived a Boltzmann-like transport

equation for quasiparticles that includes virial corrections to the scattering integral via set of shifts in time, space, momentum and energy. We have been able to proof conservation laws for density, momentum and energy [9,10]. The presented theory extends the theory of quantum gases [11,12] and non-ideal plasma [13] to degenerated system.

With respect to numerical implementations the presented theory is as simple as possible: the scattering integral (2) includes only six-dimensional integration as the standard BE, the virial corrections in form of Δ's are friendly to simulation Monte Carlo methods. Numerical tractability of the presented transport equation documents Ref. [14], where space shifts estimated from ground state have been used.

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